Impact of Personnel Allocation on Deterministic Planning and Scheduling

T. Majozi and (Frank) X. X. Zhu
Department of Process Integration, University of Manchester Institute of Science and Technology,
Manchester M60 1QD, United Kingdom

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The effect of personnel or operator allocation on batch plant scheduling is presented. The main challenge in dealing with operator allocation is the fact that operators tend to be more qualitative than quantitative, which renders traditional mathematical techniques inadequate. The scheduling model presented in this article takes into consideration the competency of selected and allocated operators. Also presented in this article is a new planning model to determine optimal scheduling time horizons on normal and penalized (holiday or weekend) days, concomitant with the corresponding shift patterns. The difference in basic and overtime payment is taken into account to determine the length of working time of each operator on a given day. Associated with the length of working time is the appropriate shift pattern, given that very long hours may justify more than one shift. The resultant integrated planning and scheduling model is decomposed by exploiting its block angular structure. © 2004 American Institute of Chemical Engineers *AIChE J,* 50: 999–1016, 2004
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Introduction

The problem of batch-process scheduling has been around for more than 25 years. In simple terms, the problem concerns the determination of an optimal sequence of tasks in a given set of resources, with a view to satisfy a specific performance index. The latter could be minimization of makespan or maximization of profit, for instance. In its evolution, the problem of batch-process scheduling has assumed various forms. The initial mathematical formulations were mixed integer nonlinear programming (MINLP) models for which global optimality could not be readily guaranteed. Among these are formulations of Sparrow et al. (1975), Grossmann and Sargent (1979), and Suhami and Mah (1982). To ensure global optimality, Kondili et al. (1993) proposed a mixed integer linear programming (MILP) formulation that was based on discretization of time horizon into equal time intervals. The major drawback of this formulation is that accuracy depends on the chosen length of each time interval, which might require a large number of binary variables for some problems. As a result of this drawback, recent work has focused on developing continuous time formulations (Ierapetritou and Floudas, 1998; Majozi and Zhu, 2001; Mockus and Reklaitis, 1997; Pinto and Grossmann, 1995; Schilling and Pantelides, 1996; Zhang and Sargent, 1994). Moreover, advances in the field of batch process scheduling have shown that this problem is better addressed in conjunction with production planning (Bassett et al., 1996; Orçun et al., 2001; Rodrigues et al., 2000; Subrahmanyam et al., 1996; Zhu and Majozi, 2001). However, all these developments have either inadequately addressed or completely ignored some of the intrinsic features of batch operations.

The performance of batch chemical processes is dependent on a variety of issues emanating from a cohort of uncertainties that are intrinsic in batch operations. Although most of these uncertainties are quantitative in nature (such as purity of raw materials, activity or age of catalyst, temperature and pressure profiles), some tend to be qualitative. Most imperative among the latter is the quality of the operators allocated to individual

Correspondence concerning this article should be addressed to X. X. Zhu at Frank.Zhu@uop.com.

plants. Human intervention in batch chemical plants is one of the significant causes for variations in plant performance by different crews (Weaver and Bauman, 1984). This evolves from differences in expertise, experience, and skills, for example. Lengthy time horizons required by production plans demand that some plants have to operate more shifts than one, and each shift requires a different crew—thus the variations in plant performance. In their recent publication, Orçun et al. (2001) highlight the effect of different operator skills on planning and scheduling. Their approach is based on modular representation of operations, where an operation (module) is composed of several operators performing the same task. In the context of their methodology, operators refer to both equipment and personnel. The generality of the formulation allows variations in performance for the operators within the same module. However, the performance of each operator is fixed throughout the planning and scheduling time horizon. As a result, the issue of different crews employed in a given operation within the planning and scheduling time horizon, which culminates in different performances for the same equipment, is completely overlooked. Another challenge in dealing with operators arises from the fact that the qualitative features that tend to adequately characterize operator performance (such as age, skill, expertise, health, and so on) cannot be easily put into mathematical equations. In the face of this situation, production planners and schedulers resort to heuristic procedures, with more emphasis on feasibility than optimality (Rodrigues et al., 2000). Although it cannot be argued that operator performance has an impact on the overall plant performance (Honkomp et al., 2000; Shobrys and White, 2000), almost all the work on planning and scheduling, which has gained much popularity in the last decade, has completely ignored this issue.

In this article, the interaction between the quality of the operators and the structure of the operation schedule is explored. This concept is motivated by the fact that the operation schedule is dependent on the performance of unit operations, which is in turn influenced by the quality of operators as mentioned earlier. Complication on this issue arises from establishing an ideal match between what the plant requires and operator capabilities. A method to establish this match involves the determination of the grades of each operator and the grade required in each plant using fuzzy set theory as presented by Majozi and Zhu (2004). There are two types of days: normal and penalized. In the context of this article, a day that is not a holiday or weekend is classified as normal; otherwise, the day is classified as penalized. Moreover, the issue of remuneration rates has an impact on the length of time worked on a given day. Although favorable for production, long hours might justify overtime, which is concomitant with high remuneration rates. This might not be financially viable. To avoid this, a concept of two- and three-shift patterns is usually adopted. This concept is also motivated by the fact that work regulations (labor laws) prevalent in each country usually set a limit on the length of overtime allowed per day. Also embedded within the issue of remuneration rates is the optimal number of operators. In this era of industrial downsizing, emphasis on zero redundancy, which is congruent to the optimal number of operators, has surfaced as one of the most prominent issues. In this regard, there certainly is a need for the development of a technically justified procedure for the selection of required and rejection of redundant personnel. The concern about the optimal number of operators presents further challenges. First, the selection and allocation of operators is usually treated as a planning decision, whereas the optimal number of operators in batch operations is dependent on the number of active units according to the production schedule. Therefore, an overall optimal number of operators can be achieved only by integrating the planning and scheduling levels. Second, the determination of the optimal number of personnel is affected by personnel scheduling. The latter is one of the classic problems in operations research known as the *labor tour scheduling* problem, concerned with the assignment of personnel to individual unit operations without violating legal requirements, which includes the number of hours worked by each operator over the entire planning time horizon, number of days off, number of part-time and full-time employees, and so forth. This concept is beyond the scope of the material presented in this article. It is worthy of mention, however, that even tour scheduling problems are usually based on the assumption that duties to be performed by personnel are available throughout the planning or scheduling time horizon (Ashley, 1995; Aykin, 2000; Beaumont, 1997; Brusco and Johns, 1996; Lagodimos and Leopoulos, 2000; Lin et al., 2000). This renders them more appropriate to continuous than batch operations.

All the issues raised in the foregoing discussion inspired the content of the material presented in this article. A novel planning model that determines the optimal scheduling time horizons for any given plant on normal and penalized days is developed. Together with the optimal scheduling time horizons are the optimal shift patterns and optimal allocation of operators and raw materials to individual plants. The integration of this model with the scheduling models for individual plants through the exploitation of the block angular structure, exhibited by most integrated planning and scheduling models (Zhu and Majozi, 2001), ensures an overall optimal number of operators. The planning model is initially formulated as an MINLP with nonlinearities arising from the bilinear terms in the cost constraints. These are linearized using known mathematical techniques to yield and MILP model.

Problem Statement

In the context of this article, an integrated planning and scheduling problem can be stated as follows. Given:

(1) rank or grade of each operator and the available number to choose from (selection pool),

(2) the number of available plants and the rank or grade required in each plant,

(3) baseline/overtime remuneration for each operator on normal and penalized days,

(4) number of operators required in each stage/unit of the given plants,

(5) the cost of raw materials and product selling price,

(6) effluent and waste disposal costs,

(7) stoichiometric relations between raw materials and products,

(8) the production recipe for each product, including mean processing times in each unit operation,

(9) the available units and their capacities,

(10) the maximum storage capacity for each material, and

(11) the number of penalized and normal days over the planning time horizon of interest,

determine:

(1) the optimal number of operators in each plant,

(2) the optimal allocation of operators to different plants,

(3) the length of scheduling time horizon on normal and penalized days,

(4) the appropriate shift pattern in each plant on normal and penalized days,

(5) the personnel requirement in each plant at each point along the scheduling time horizon,

(6) raw material requirement on normal and penalized days,

(7) product throughput on normal and penalized days for maximum profit,

(8) the optimal schedule for tasks within the time horizon of interest,

(9) the amount of material processed at any particular point in time within the scheduling time horizon, and

(10) the amount delivered to customers over the scheduling time horizon.

Mathematical Model

Before delving into details of the mathematical formulation, it is worth elaborating on the assumptions made in this article. These relate mainly to scheduling time horizon and shift patterns, determined at the planning level. On any given day each operator is allowed to work only one shift, which spans a maximum of 12 h. This corresponds to maximum regular-time (baseline) and overtime of 8 and 4 h, respectively. Therefore, if a given shift requires more than 8 h, any time beyond 8 h is treated as overtime, and should not exceed 4 h. If a particular plant requires a scheduling time horizon of more than 12 h on a given day (normal or penalized), then a two- or three-shift pattern has to be selected, depending on whether the shift ends before or after 16 h, respectively. However, even a three-shift pattern should not exceed 24 h. Also, the length of individual shifts corresponding to two- and three-shift patterns is equal. For example, if an optimal scheduling time horizon is 14 h, then each of the corresponding shifts will span over 7 h. Depending on optimality, scheduling time horizons of exactly 12 and 16 h can either correspond to single or two-shift and two- or three-shift patterns, respectively. In a situation where the scheduling time horizon is 12 h, it should be noted that employing few people, and remunerating them for overtime, as required by the single-shift pattern, could be more (less) expensive than employing more people in a two-shift pattern (each spanning over 6 h) without overtime. The same argument also holds in a situation where the scheduling time horizon is 16 h and the choice has to be made between two- and three-shift patterns. It has also been assumed that the minimum and maximum grades for the operators are 8 and 18, respectively, with remuneration increasing linearly with the grade. It is worth mentioning at this stage that different grading schemes will have different grade-to-remuneration correspondence. However, this issue is no cause for concern because these are provided as parameters in the planning model.

The features of the planning model presented by Zhu and Majozi (2001) still hold in this article. For the uninitiated, these relate to raw materials, products, and by-products. Below a specific amount, raw materials are available at a standard cost. Beyond this amount, penalties are imposed, which is a common encounter where a new supplier has to be approached to meet an unexpected surge in demand. Moreover, any raw material can be shared by different plants. Because the manufacture of most products involves by-product formation, this model also includes by-product disposal costs. This cost is accompanied by penalties when by-product production exceeds a stipulated amount, as commonly encountered in industries that produce highly toxic material. These penalties, which are usually enacted by local or national authorities, provide a motive to discourage excess production of toxic material. Also, each product is characteristic of a particular process, as usually encountered in high value products, as a means of ensuring product integrity. However, as the distinct feature of multipurpose batch plants, different batches of the same product can follow different routes within the same process.

The overall mathematical model entails both planning and scheduling models as detailed below. The mathematical symbols used are described in the nomenclature.

Planning Model

Although the planning model presented in this section is novel, some of its features are very similar to the model presented by Zhu and Majozi (2001). It should be noted that the expressions corresponding to normal and penalized days have been separated throughout the planning model. Although this makes the model look complicated, it certainly improves computational performance by avoiding tri-index binary variables.

Raw material availability constraints

$$W(s_{in}^r) - W'(s_{in}^r) \le R(s_{in}^r) \qquad \forall\ s_{in}^r \in S_{in}^r \tag{1}$$

$$\sum_k w(s_{in,k}^r) = W(s_{in}^r) \qquad \forall\ s_{in}^r \in S_{in}^r \tag{2}$$

$$W'(s_{in}^r) \le R'(s_{in}^r) \qquad \forall\ s_{in}^r \in S_{in}^r \tag{3}$$

$$w(s_{in,k}^r) = wn(s_{in,k}^r)Dn(k) + wp(s_{in,k}^r)Dp(k) \qquad \forall\ s_{in,k}^r \in S_{in,k}^r,\ k \in K \tag{4}$$

$$W'(s_{in}^r) \ge 0,\ W(s_{in}^r) \ge 0,\ w(s_{in,k}^r) \ge 0,\ wn(s_{in,k}^r) \ge 0,\ wp(s_{in,k}^r) \ge 0 \tag{5}$$

Constraint 1 states that the amount of raw material s_{in}^r available at standard price is the difference between the overall amount used and the amount available at penalized cost. Constraint 2 denotes that the overall amount of raw material s_{in}^r used is the sum of all the amounts of raw material s_{in}^r used in different processes k. Constraint 3 sets an upper bound on the amount available at penalized cost. In this article, the upper bound on the amount of penalized raw materials is arbitrarily chosen as a very large number. Constraint 4 states that the overall amount of raw material used over the entire planning time horizon is the sum of the amounts used on normal and penalized days.

Product constraints

$$w(s^d_{out,k}) = wn(s^d_{out,k})Dn(k) + wp(s^d_{out,k})Dp(k) \qquad \forall\ s^d_{out,k} \in S^d_{out,k}, k \in K \quad (6)$$

$$wn(s^d_{out,k}) \le wn^U(s^d_{out,k}) \qquad \forall\ s^d_{out,k} \in S^d_{out,k}, k \in K \quad (7)$$

$$wp(s^d_{out,k}) \le wp^U(s^d_{out,k}) \qquad \forall\ s^d_{out,k} \in S^d_{out,k}, k \in K \quad (8)$$

$$wn^U(s^d_{out,k}) = Grad(k)\sum_{l\in L} Hn(k, l) + Yin(k)\sum_{l\in L} sn(k, l) \qquad \forall\ s^d_{out,k} \in S^d_{out,k}, k \in K \quad (9)$$

$$wp^U(s^d_{out,k}) = Grad(k)\sum_{l\in L} Hp(k, l) + Yin(k)\sum_{l\in L} sp(k, l) \qquad \forall\ s^d_{out,k} \in S^d_{out,k}, k \in K \quad (10)$$

$$wn(s^d_{out,k}) \ge wn^L(s^d_{out,k})\sum_{l\in L} sn(k, l) \qquad \forall\ s^d_{out,k} \in S^d_{out,k}, k \in K \quad (11)$$

$$wp(s^d_{out,k}) \ge wp^L(s^d_{out,k})\sum_{l\in L} sp(k, l) \qquad \forall\ s^d_{out,k} \in S^d_{out,k}, k \in K \quad (12)$$

$$w(s^d_{out,k}) \ge 0, w(s^d_{out,k}), wn(s^d_{out,k}) \ge 0, wp(s^d_{out,k}) \ge 0 \quad (13)$$

Constraint 6 states that the amount of product produced over the planning time horizon is the sum of the amounts produced on normal and penalized days. There is always an upper bound on the amount that can be produced per day, depending on unit capacities and time horizons on normal and penalized days, as represented by constraints 7 and 8, respectively. Constraints 9 and 10 relate the upper bounds on production for normal and penalized days to corresponding time horizons. These equations are derived from regression analysis. It was realized that for all the problems considered, production throughput increased linearly with the time horizon. This is demonstrated later in this article. However, the universality of this linearity has not been proven yet. According to constraints 11 and 12, there is a lower bound on the amount of product that can be produced on a particular day. Collectively, constraints 9 and 10 and constraints 11 and 12 ensure that, if no shift pattern is selected for a particular plant on any particular day, then the plant should not operate on that particular day.

By-product constraints

$$W(s^b_{out}) - W'(s^b_{out}) \le B(s^b_{out}) \qquad \forall\ s^b_{out} \in S^b_{out} \quad (14)$$

$$\sum_k w(s^b_{out,k}) = W(s^b_{out}) \qquad \forall\ s^b_{out} \in S^b_{out} \quad (15)$$

$$W'(s^b_{out}) \le B'(s^b_{out}) \qquad \forall\ s^b_{out} \in S^b_{out} \quad (16)$$

$$w(s^b_{out,k}) = wn(s^b_{out,k})Dn(k) + wp(s^b_{out,k})Dp(k) \qquad \forall\ s^b_{out,k} \in S^b_{out,k}, k \in K \quad (17)$$

$$W(s^b_{out}) \ge 0, \qquad W'(s^b_{out}) \ge 0, \qquad w(s^b_{out,k}) \ge 0 \quad (18)$$

The explanation for constraints 14 to 17 is similar to that for constraints 1 to 4.

Stoichiometric constraints

$$\beta(s^b_{out,k}) = \frac{wn(s^b_{out,k})}{wn(s^d_{out,k})} \qquad \forall\ k \in K, s^b_{out,k} \in S^b_{out,k}, s^d_{out,k} \in S^d_{out,k} \quad (19)$$

$$\beta(s^b_{out,k}) = \frac{wp(s^b_{out,k})}{wp(s^d_{out,k})} \qquad \forall\ k \in K, s^b_{out,k} \in S^b_{out,k}, s^d_{out,k} \in S^d_{out,k} \quad (20)$$

$$\alpha(s^r_{in,k})\left(wn(s^d_{out,k}) + \sum_{s^b_{out,k}} wn(s^b_{out,k})\right) = wn(s^r_{in,k}), \qquad \forall\ k \in K, s^r_{in,k} \in S^r_{in,k}, s^d_{out,k} \in S^d_{out,k}, s^b_{out,k} \in S^b_{out,k} \quad (21)$$

$$\alpha(s^r_{in,k})\left(wp(s^d_{out,k}) + \sum_{s^b_{out,k}} wp(s^b_{out,k})\right) = wp(s^r_{in,k}), \qquad \forall\ k \in K, s^r_{in,k} \in S^r_{in,k}, s^d_{out,k} \in S^d_{out,k}, s^b_{out,k} \in S^b_{out,k} \quad (22)$$

Constraints 19 and 20 are the stoichiometric relationships between the amount of product $s^d_{out,k}$ produced and the corresponding by-product $s^b_{out,k}$ produced from plant k on normal and penalized days, respectively. Constraints 21 and 22 are the stoichiometric relationships between the amount of raw material $s^r_{in,k}$ used in process k and the overall output, including both product and related by-products from process k on normal and penalized days, respectively.

Scheduling time horizon and shift pattern constraints

$$Hn^L(l)sn(k, l) \le Hn(k, l) \le Hn^U(l)sn(k, l) \qquad k \in K, l \in L \quad (23)$$

$$Hn(k, 1) = Hn^{ov}sn(k, 1) + Ovtn(k) \qquad k \in K \quad (24)$$

$$0 \le Ovtn(k) \le [Hn^U(1) - Hn^{ov}]sn(k, 1) \qquad k \in K \quad (25)$$

$$Hn(k, l) = Hn^L(l)sn(k, l) + \mathrm{Varn}(k, l) \qquad k \in K, l = 2, 3, l \in L \quad (26)$$

$$0 \le Varn(k, l) \le [Hn^U(l) - Hn^L(l)]sn(k, l) \qquad k \in K, l = 2, 3, l \in L \quad (27)$$

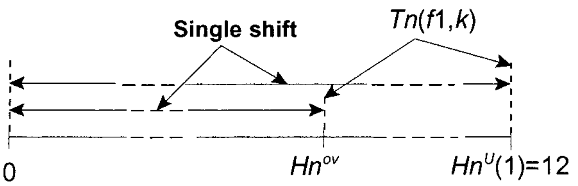

Figure 1. Single-shift pattern representation.

$$\sum_{l \in L} sn(k, l) \le 1 \qquad k \in K \tag{28}$$

$$Hp^L(l)sp(k, l) \le Hp(k, l) \le Hp^U(l)sp(k, l) \qquad k \in K, l \in L \tag{29}$$

$$Hp(k, 1) = Hp^{ov}sp(k, 1) + Ovtp(k) \qquad k \in K \tag{30}$$

$$Hp(k, l) = Hp^L(l)sp(k, l) + Varp(k, l) \qquad k \in K, l = 2, 3, l \in L \tag{31}$$

$$0 \le Ovtp(k) \le [Hp^U(1) - Hp^{ov}]sp(k, 1) \qquad k \in K \tag{32}$$

$$0 \le Varp(k, l) \le [Hp^U(l) - Hp^L(l)]sp(k, l) \qquad k \in K, l = 2, 3, l \in L \tag{33}$$

$$\sum_{l \in L} sp(k, l) \le 1 \qquad k \in K \tag{34}$$

$$sn(k, l), sp(k, l) = 0/1 \tag{35}$$

Constraints 23 to 28 and 29 to 34 correspond to normal day and penalized day time horizons, respectively. Constraint 23 sets upper and lower bounds on the normal day time horizon. Constraint 24 states that the time horizon corresponding to a single-shift pattern constitutes baseline time and overtime. Constraint 25 states that the upper bound on the length of overtime in any particular plant (k) is the difference between the maximum length of the single-shift pattern and baseline time. Otherwise, the time horizon is composed of the minimum length and variable time, as given in constraint 26. The upper bound on variable time is the difference between the maximum and the minimum length of the corresponding shift pattern, as shown in constraint 27. According to constraint 28, at most one shift pattern can be allocated to a particular plant on a normal day. This implies that a plant can operate only on a single-shift, two-shift, or three-shift pattern on any normal day. Constraints 29 to 34 are congruent to constraints 23 to 28.

Figures 1, 2, and 3 represent single-, two-, and three-shift patterns, respectively. In the context of this article, the maximum length of a single-shift pattern has been chosen as 12 h. The latter also corresponds to the minimum length of a two-shift pattern for which the maximum length has been chosen as 16 h. The minimum and maximum lengths for the three-shift pattern are 16 and 24 h, respectively.

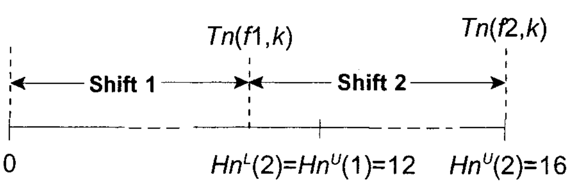

Figure 2. Two-shift pattern representation.

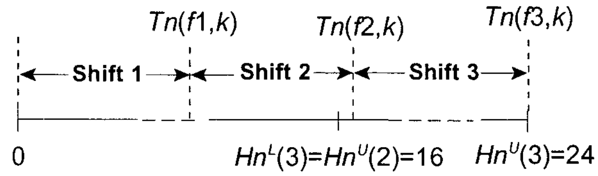

Figure 3. Three-shift pattern representation.

Allocation constraints

$$xn(o, k)[G(k) - G'(o)] + \delta n^-(o, k) - \delta n^+(o, k) = 0 \qquad k \in K, o \in O \tag{36}$$

$$xp(o, k)[G(k) - G'(o)] + \delta p^-(o, k) - \delta p^+(o, k) = 0 \qquad k \in K, o \in O \tag{37}$$

$$\sum_{o \in O} xn(o, k) = Nn^p(k) \qquad k \in K \tag{38}$$

$$\sum_{o \in O} xp(o, k) = Np^p(k) \qquad k \in K \tag{39}$$

$$Nn^p(k) = \sum_{l \in L} On(k, l)sn(k, l) \qquad k \in K \tag{40}$$

$$Np^p(k) = \sum_{l \in L} Op(k, l)sp(k, l) \qquad k \in K \tag{41}$$

$$\sum_{k \in K} xn(o, k) \le 1 \qquad o \in O \tag{42}$$

$$\sum_{k \in K} xp(o, k) \le 1 \qquad o \in O \tag{43}$$

$$\delta n^+(o, k), \delta n^-(o, k), \delta p^+(o, k), \delta p^-(o, k) \ge 0 \tag{44}$$

$$xn(o, k), xp(o, k) = 0/1 \tag{45}$$

Constraints 36 and 37 use slack variables to provide for deviation between the rank required in plant k and the rank of a particular operator o. The ultimate goal is to maximize the slack variables providing for negative deviation, which is similar to selecting high-grade operators, thereby maximizing plant performance. It is worthy of note that there is an allowance for one operator to work in different plants, depending on whether it is a normal or penalized day. Constraints 38 and 39 state that the number of operators allocated to each plant should be equal to the number required in each plant on normal and penalized days, respectively. The number of operators required in each plant is obtained according to constraints 40 and 41 for normal and penalized days, respectively. $On(k, l)$ and $Op(k, l)$ are obtained from the solution of the scheduling models, and used in these constraints as parameters. However, for the initial solution (that is, before solving the scheduling models), the number of operators required in a particular plant k for a particular shift pattern l is based on the operator requirement for each unit. This is explained further in subsequent sections of this article. Constraints 42 and 43 state that each operator can be allocated at most to only one plant on a given day. This implies that in a given selection pool, some of the operators might be omitted, as long as the operator requirement for each plant is met.

Cost constraints

Single-Shift Pattern

$$SPC(k, 1) = \left[Hn^{ov} sn(k, 1) \sum_{o \in O} Rem(o, m1) xn(o, k) + Ovtn(k) \sum_{o \in O} Rem(o, m2) xn(o, k) \right] Dn(k) + \left[Hp^{ov} sp(k, 1) \sum_{o \in O} Rem(o, m3) xp(o, k) + Ovtp(k) \sum_{o \in O} Rem(o, m4) xp(o, k) \right] Dp(k) \qquad k \in K \quad (46)$$

Two- and Three-Shift Patterns

$$SPC(k, l) = \left[Hn^{L}(l) sn(k, l) \sum_{o \in O} Rem(o, m1) xn(o, k) + \mathrm{Var}\, n(k, l) \sum_{o \in O} Rem(o, m1) xn(o, k) \right] Dn(k) + \left[Hp^{L}(l) sp(k, l) \sum_{o \in O} Rem(o, m3) xp(o, k) + \mathrm{Var}\, p(k, l) \sum_{o \in O} Rem(o, m3) xp(o, k) \right] Dp(k) \qquad k \in K, l = 2, 3, l \in L \quad (47)$$

According to constraint 46, the cost corresponding to a single-shift pattern in a particular plant (k) over the entire planning time horizon is composed of:

(1) the cost associated with remunerating operator o during baseline time on normal days ($m1$),

(2) the cost associated with remunerating operator o during overtime on normal days ($m2$),

(3) the cost associated with remunerating operator o during baseline time on penalized days ($m3$), and

(4) the cost associated with remunerating operator o during overtime on penalized days ($m4$).

This constraint makes an allowance for baseline and overtime costs to vary according to day type, either normal or penalized.

Constraint 47 states that the cost corresponding to two- and three-shift patterns in a particular plant (k) over the entire planning time horizon is composed of:

(1) the cost associated with remunerating operator o during baseline time on normal days ($m1$) and

(2) the cost associated with remunerating operator o during baseline time on penalized days ($m3$).

It is worthy of note that in two- and three-shift patterns there is no opportunity for overtime. This is attributed to the fact that, in the context of this article, operators are not allowed to work more than one shift on a given day.

Constraints 46 and 47 consist of bilinear terms, which impose nonlinearity in the overall model. However, because these arise only from the product of either two binary variables or a binary variable and a continuous variable, they can be linearized using Glover transformation (1975) as shown below. An attempt was made to apply the Watters transformation (1967) for linearization of the former set of bilinear terms (that is, two binary variables). However, this requires an additional binary variable, which increases the computational effort in solving the model.

Linearization (Glover Transformation). Let:

$$sn(k, l) xn(o, k) = Gn^{l}(o, k)$$
$$xn(o, k) Ovtn(k) = Gn^{1,ovtn}(o, k)$$
$$xn(o, k) Varn(k, l) = Gn^{l,varn}(o, k)$$
$$sp(k, l) xp(o, k) = Gp^{l}(o, k)$$
$$xp(o, k) Ovtp(k) = Gp^{1,ovtp}(o, k)$$
$$xp(o, k) Varp(k, l) = Gp^{l,varp}(o, k)$$

then,

$$sn(k, l) - [1 - xn(o, k)] \le Gn^{l}(o, k) \le sn(k, l) \quad (48)$$

$$0 \le Gn^{l}(o, k) \le xn(o, k) \quad (49)$$

$$Ovtn(k) - [Hn^{U}(1) - Hn^{ov}][1 - xn(o, k)] \le Gn^{1,ovtn}(o, k) \le Ovtn(k) \quad (50)$$

$$0 \le Gn^{1,ovtn}(o, k) \le [Hn^{U}(1) - Hn^{ov}] xn(o, k) \quad (51)$$

$$Varn(k, l) - [Hn^{U}(l) - Hn^{L}(l)][1 - xn(o, k)] \le Gn^{l,\mathrm{varn}}(o, k) \le Varn(k, l) \quad (52)$$

$$0 \le Gn^{l,\mathrm{varn}}(o, k) \le [Hn^{U}(l) - Hn^{L}(l)] xn(o, k) \quad (53)$$

$$sp(k, l) - [1 - xp(o, k)] \le Gp^{l}(o, k) \le sp(k, l) \quad (54)$$

$$0 \le Gp^{l}(o, k) \le xp(o, k) \quad (55)$$

$$Ovtp(k) - [Hp^{U}(1) - Hp^{L}(1)][1 - xp(o, k)] \le Gp^{1,ovtn}(o, k) \le Ovtp(k) \quad (56)$$

$$0 \le Gp^{1,ovtn}(o, k) \le [Hp^{U}(1) - Hp^{L}(1)] xp(o, k) \quad (57)$$

$$Varp(k,l) - [Hp^U(l) - Hp^L(l)][1 - xp(o,k)] \le Gp^{l,varp}(o,k) \le Varp(k,l) \quad (58)$$

$$0 \le Gp^{l,\mathrm{var}\,p}(o,k) \le [Hp^U(l) - Hp^L(l)]xp(o,k) \quad (59)$$

The following constraint associates the remuneration of each operator to the corresponding rank. It should be noted that remuneration is dependent on the pay category. The pay categories are:

- pay corresponding to baseline time on a normal day (*m*1),
- pay corresponding to overtime on a normal day (*m*2),
- pay corresponding to baseline time on a penalized day (*m*3), and
- pay corresponding to overtime on a penalized day (*m*4).

Remuneration has been assumed to be a linear function of the rank as shown in constraint 60. However, the type of remuneration–rank function does not affect the linearity of the overall model because remuneration is supplied as a parameter. In reality, this function is dependent on the type of remuneration pattern or scale adopted by individual companies

$$Rem(o,m) = \left[\frac{Rem^U(m) - Rem^L(m)}{G^U(o) - G^L(o)}\right][G'(o) - G^L(o)] + Rem^L(m) \qquad o \in O, m \in M \quad (60)$$

Objective Function

The objective function is the maximization of profit, which takes into account the quality of operator allocation and costs. To provide for the quality of operator allocation, the penalty factor (Λ) is used to ensure a minimum possible deviation between the rank of an operator and that required by the plant. The use of the penalty factor subjects the formulation to the main drawback of penalty functions; that is, the optimal value depends on the value of Λ. Therefore, the value of this factor has to be reported for every set of results. The ranks are a product of an evaluation procedure presented in an article by Majozi and Zhu (2002). It is worthy of note that the objective function is seeking to maximize the slack variables providing for negative deviation (which corresponds to the selection of high rank operators) while minimizing the operator costs

$$\max Z = \sum_{s_{out}^d} C(s_{out}^d)W(s_{out}^d) - \sum_{s_{out}^b} C(s_{out}^b)[W(s_{out}^b) - W'(s_{out}^b)] - \sum_{s_{in}^r} C(s_{in}^r)[W(s_{in}^r) - W'(s_{in}^r)] - \sum_{s_{out}^b} C'(s_{out}^b)W'(s_{out}^b) - \sum_{s_{in}^r} C'(s_{in}^r)W'(s_{in}^r) - \sum_l \sum_k SPC(k,l) + \Lambda\left\{\sum_o \sum_k [\delta n^-(o,k) + \delta p^-(o,k) - \delta n^+(o,k) - \delta p^+(o,k)]\right\} \quad (61)$$

The solution of the planning model yields the essential data for the determination of the parameters used in the duration constraints for the scheduling model [that is, $\rho n^+(s,p)$, $\rho n^-(s,p)$, $\rho p^+(s,p)$, and $\rho p^-(s,p)$]. In the initial presentation of the scheduling model (Majozi and Zhu, 2001), these parameters were introduced as variables to provide for added degrees of freedom commonly encountered in batch operations. It was stated that these could be attributable to choice of catalyst and raw materials, as well as human intervention. The latter is ascribed to the fact that different operators tend to have different skills, expertise, and plant understanding, manifest in variable plant performance by different crews. The concept of added degrees of freedom was introduced to avoid associating the variation in batch times with the variation in batch sizes. It should be emphasized, however, that although these variables were originally presented as also providing for the choice of catalyst and raw materials, only the human intervention aspect is treated in this article. The following section demonstrates the determination of these variables, which are later incorporated as parameters into the scheduling model.

Determination of $\rho n^+(s,p)$ and $\rho n^-(s,p)$

These parameters were initially introduced as variables to provide added degrees of freedom (Majozi and Zhu, 2001), as shown in constraint 62. They are hereby introduced as parameters determined by $\delta n^+(o,k)$ and $\delta n^-(o,k)$ from the planning model

$$tn_p(s_{out,j},p) = tn_u(s^*_{in,j},p-1) + \tau(s^*_{in,j})yn(s^*_{in,j},p-1) + \rho n^+(s^*_{in,j},p-1) + \rho n^-(s^*_{in,j},p-1) \qquad \forall\ p \in P_{kn}, j \in J_k \quad (62)$$

$$\rho n^+(s^*_{in,j},p) \ge 0,\ \rho n^-(s^*_{in,j},p) \le 0 \qquad \forall\ p \in P_{kn}, j \in J_k \quad (63)$$

To decipher the mechanism on which the link between these parameters and variables is based, it is worthy of note that $\rho n^+(s,p)$ is associated with longer processing times, whereas $\rho n^-(s,p)$ is associated with shorter processing times. Therefore, it is justified to conclude that $\rho n^+(s,p)$ is characteristic of poor plant performance, whereas $\rho n^-(s,p)$ is characteristic of good plant performance. In the context of this article, this performance is influenced only by the quality (competence) of the operators allocated to each plant. This implies that the worst and best possible allocations will give maximum and minimum (most negative) values of $\rho n^+(s,p)$ and $\rho n^-(s,p)$, respectively. The latter are usually known beforehand and based on experience.

Determination of $\rho n^-(s,p)$

This parameter is associated with good plant performance, which will be realized if the rank of each operator in a given plant is greater than or equal to the rank required in that plant [i.e., $G(k) - G'(o) \le 0$]. However, it is worthy of note that, although allocating an operator whose rank is strictly greater than that required in a plant might be advantageous in terms of plant performance, it might not be economically viable, given that remuneration is proportional to the rank. To determine $\rho n^-(s,p)$, the following model has to be solved.

Model M1

$$\text{Max} \sum_{o \in O} \sum_{k \in K} [\delta n^{-}(o, k) - \delta n^{+}(o, k)] \tag{64}$$

subject to

$$\sum_{o \in O} xn(o, k) = \sum_{l \in L} O^{max}(k, l) sn(k, l) \qquad k \in K \tag{65}$$

$$\sum_{k \in K} xn(o, k) \leq 1 \qquad o \in O \tag{66}$$

$$xn(o, k) = 0/1 \tag{67}$$

According to the objective function (Eq. 64), this model is seeking to maximize the sum of slack variables to provide for negative deviation between the rank required in each plant and the ranks of the operators allocated to the plant. The binary variable associated with the choice of a shift pattern, that is, $sn(k, l)$, used in constraint 65 is obtained directly from the planning model output. Therefore, in this constraint, $sn(k, l)$ is used as a parameter. Equation 68 is the maximum sum of negative deviation slack variables, which is obtained after solving model M1.

$$\max \sum_{o} \sum_{k} \delta n^{-}(o, k) = \left[\sum_{o} \sum_{k} \delta n^{-}(o, k) \right]_{M1} \tag{68}$$

To relate $\rho n^{-}(s, p)$ to the maximum sum of negative deviation slack variables, a ratio between the actual and maximum sum of negative deviation slack variables is defined as follows

$$\Delta n^{-}(k) = \frac{[\sum_{o \in O} \sum_{k \in K} \delta n^{-}(o, k)]^{actual}}{[\sum_{o \in O} \sum_{k \in K} \delta n^{-}(o, k)]_{M1}} \tag{69}$$

The actual sum of negative deviation slack variables is obtained from the output of the planning model. It is worthy of note that the maximum value of $\Delta n^{-}(k)$ (that is, 1) represents operator allocation wherein the ranks of most or all operators are higher than the rank required in a particular plant k. This is likely to culminate in best plant performance. Therefore, it is conjectured that $\rho n^{-}(s, p)$ is, in one way or another, dependent on $\Delta n^{-}(k)$ {that is, $\rho n^{-}(s, p) = f[\Delta n^{-}(k)]$}. A maximum value of $\Delta n^{-}(k)$ (that is, 1) corresponds to the minimum (most negative) value of $\rho n^{-}(s, p)$, and a minimum value of $\Delta n^{-}(k)$ (that is, 0) corresponds to the maximum value of $\rho n^{-}(s, p)$ (that is, 0). For a unit operation in which the allowed processing time variation is ν and the mean processing time is $\tau(s)$, the minimum value of $\rho n^{-}(s, p)$ is $-\nu\tau(s)$. Assuming a linear correspondence between $\rho n^{-}(s, p)$ and $\Delta n^{-}(k)$ gives the following equation

$$\rho n^{-}(s^{*}_{in,j}, p) = [-\nu\tau(s^{*}_{in,j})]\Delta n^{-}(k) \qquad j \in J_k, p \in P \tag{70}$$

This is the parameter that is used in Eq. 62.

Determination of $\rho n^{+}(s, p)$

This parameter is associated with poor plant performance, which will be realized if the rank of each operator allocated to a given plant is less than the rank required in that plant [i.e., $G(k) - G'(o) > 0$]. It is determined by solving the following model.

Model M2

$$\min \sum_{o} \sum_{k} [\delta n^{-}(o, k) - \delta n^{+}(o, k)] \tag{71}$$

subject to

Constraints 65, 66, and 67.

Model M2 is aimed at maximizing the sum of slack variables to provide for the positive deviation between the rank required in each plant and the rank of the operators allocated to the plant. The maximum sum of positive deviation slack variables, obtained after solving model M2, is given in the following equation

$$\max \sum_{o} \sum_{k} \delta n^{+}(o, k) = \left[\sum_{o} \sum_{k} \delta n^{+}(o, k) \right]_{M2} \tag{72}$$

The ratio between the actual and maximum sum of positive deviation slack variables is then defined as shown in Eq. 73. The actual maximum sum of positive deviation slack variables is also obtained from the output of the planning model

$$\Delta n^{+}(k) = \frac{[\sum_{o \in O} \sum_{k \in K} \delta n^{+}(o, k)]^{actual}}{[\sum_{o \in O} \sum_{k \in K} \delta n^{+}(o, k)]_{M2}} \tag{73}$$

The maximum value of $\Delta n^{+}(k)$ (that is, 1) represents operator allocation wherein the ranks of most or all operators are lower than the rank required in a particular plant k, which is likely to result in poor plant performance. Therefore, there has to be relationship between $\rho n^{+}(s, p)$ and $\Delta n^{+}(k)$. A maximum value of $\Delta n^{+}(k)$ (that is, 1) corresponds to the maximum value of$\rho n^{+}(s, p)$, and the minimum value of $\Delta n^{+}(k)$ (that is, 0) corresponds to the minimum value of $\rho n^{+}(s, p)$ (that is, 0). If the allowed variation in processing time is ν and the mean processing time is $\tau(s)$, the maximum value of $\rho n^{+}(s, p)$ is $\nu\tau(s)$. Assuming a linear relationship between $\rho n^{+}(s, p)$ and $\Delta n^{+}(k)$ results in Eq. 74.

$$\rho n^{+}(s^{*}_{in,j}, p) = [\nu\tau(s^{*}_{in,j})]\Delta n^{+}(k) \qquad j \in J_k, p \in P_{kn} \tag{74}$$

This parameter is then used in Eq. 62. It should be emphasized that models M1 and M2 are solved only once, given that the sum of maximum deviation slack variables is fixed throughout the solution procedure.

The determination of $\rho p^{+}(s, p)$ and $\rho p^{-}(s, p)$, which correspond to penalized days, follows the same procedure.

Scheduling Model

Because of a possibility of different time horizons and shift patterns, as well as daily production rates for any given plant on normal and penalized days, each plant has to be scheduled for each day category. This is achieved by applying the extended model of Majozi and Zhu (2001) to a particular plant k on normal and penalized days. The extension arises only in the duration constraints as explained above, and thus a detailed explanation of the constraints will not be given in this article because it has been presented in articles by Majozi and Zhu (2001) and Zhu and Majozi (2001). Shown below is the scheduling model for normal days. A similar model has to be constructed for penalized days. Attention should be brought to the fact that the scheduling time horizon used in the time horizon constraints for the scheduling model is obtained from the planning model output.

Scheduling Model for a Particular Plant *k* on Normal Days

Duration constraints

These constraints are given in Eq. 62, with parameters defined in Eqs. 70 and 74.

Capacity constraints

$$V_j^{min} yn(s^*_{in,j}, p) \le \sum_{s_{in,j}} mn_u(s, p) \le V_j^{max} yn(s^*_{in,j}, p) \qquad \forall\ j \in J_k, p \in P_{kn} \quad (75)$$

Material balances

$$\sum_{s=s_{in,j}} mn_u(s, p-1) = \sum_{s=s_{out,j}} mn_p(s, p) \qquad \forall\ p \in P_{kn}, j \in J_k \quad (76)$$

$$qn_s(s, p_1) = Q_s^0(s) - mn_u(s, p_1) \qquad s \ne \text{product}, s \in S_k, p_1 = \text{starting point}, p_1 \in P_{kn} \quad (77)$$

$$qn_s(s, p) = qn_s(s, p-1) - mn_u(s, p) \qquad s = \text{feed}, s \in S_k \qquad \forall\ p \in P_{kn}, p > p_1 \quad (78)$$

$$qn_s(s, p) = qn_s(s, p-1) + mn_p(s, p) - mn_u(s, p) \qquad s \ne \text{product, feed}, s \in S_k \qquad \forall\ p \in P_{kn}, p > p_1 \quad (79)$$

$$qn_s(s, p_1) = Q_s^0(s) - dn(s, p_1) \qquad s = \text{product}, s \in S_k, p_1 = \text{starting point}, p_1 \in P_{kn} \quad (80)$$

$$qn_s(s, p) = qn_s(s, p-1) + mn_p(s, p) - dn(s, p) \qquad s = \text{product, by-product}, s \in S_k \qquad \forall\ p \in P_{kn}, p > p_1 \quad (81)$$

Sequence constraints

$$tn_u(s_{in,j}, p) \ge \sum_{s_{in,j}} \sum_{s_{out,j}} \sum_{p' \le p} tn_p(s_{out,j}, p') - tn_u(s_{in,j}, p'-1) \qquad \forall\ j \in J_k, p \in P_{kn}, s_{out,j} \in S_{out,j}, s_{in,j} \in S_{in,j} \quad (82)$$

$$tn_u(s_{in,j}, p) \ge tn_p(s_{out,j}, p) \qquad \forall\ j \in J_k, p \in P_{kn} \quad (83)$$

$$tn_u(s_{in,j}, p) \ge tn_p(s_{out,j'}, p) \qquad \forall\ j, j' \in J_k, p \in P_{kn}, s_{out,j'} = s_{in,j} \quad (84)$$

Time horizon constraints

$$tn_u(s_{in,j}, p) \le Hn(k, l) \qquad \forall\ s_{in,j} \in S_{in,j}, p \in P_{kn}, j \in J_k \quad (85)$$

$$tn_p(s_{out,j}, p) \le Hn(k, l) \qquad \forall\ s_{out,j} \in S_{out,j}, p \in P_{kn}, j \in J_k \quad (86)$$

Storage constraints

$$qn_s(s, p) \le Q^{max}(s) \qquad \forall\ s \in S_k, p \in P_{kn} \quad (87)$$

Objective function

$$\text{maximize} \sum_s \sum_p dn(s, p) \qquad s = \text{product}, p \in P_{kn} \quad (88)$$

Optimal number of operators

The optimal number of operators is dependent on the number of operators required to operate a particular unit j [$A(j)$], the number of active units over a scheduling time horizon, and the optimal shift pattern. The number of active units over a scheduling time horizon is obtained after solving a scheduling model and the optimal shift pattern is from the planning model output. Equation 89 is the expression for the optimal number of operators on a normal day and a single-shift pattern. According to this expression, the operators working on a particular unit j [$A(j)$] will be required only if this unit is active during the scheduling time horizon {that is, $\max[yn(s^*_{in,j}, p)] = 1$}; otherwise, these operators will be redundant

$$On(k, 1) = \sum_{j \in J_k} A(j) \max[yn(s^*_{in,j}, p)] \quad p \in P_{kn}, k \in K\ s^*_{in,j} \in s_{in,j} \quad (89)$$

Equation 90 is an expression for the number of operators required in a two-shift pattern. This is composed of operators required in the first shift ($f1$) and operators required in the second shift ($f2$). The number of operators that is required in the first shift in a particular plant k is dependent on the number of active units before the end of the first shift [that is, $tn_u(s^*_{in,j}, p) < Tn(f1, k)$]. The number of operators required in the second shift is dependent on the number of units that finish operating after the end of the first shift [that is, $tn_p(s_{out,j}, p) > Tn(f1, k)$]. It is worthy of note that this includes the units that start their operation in the first shift, but only finish in the second

$$On(k, 2) = \sum_{j \in J_k} A(j) \max_{tn_u(s^*_{in,j},p) < Tn(f1,k)} [yn(s^*_{in,j}, p)] + \sum_{j \in J_k} A(j) \max_{tn_p(s_{out,j},p+1) > Tn(f1,k)} [yn(s^*_{in,j}, p)]$$
$$p \in P_{kn}, \quad k \in K, \quad f1 \in F_2, \quad s^*_{in,j} \in S_{in,j} \quad (90)$$

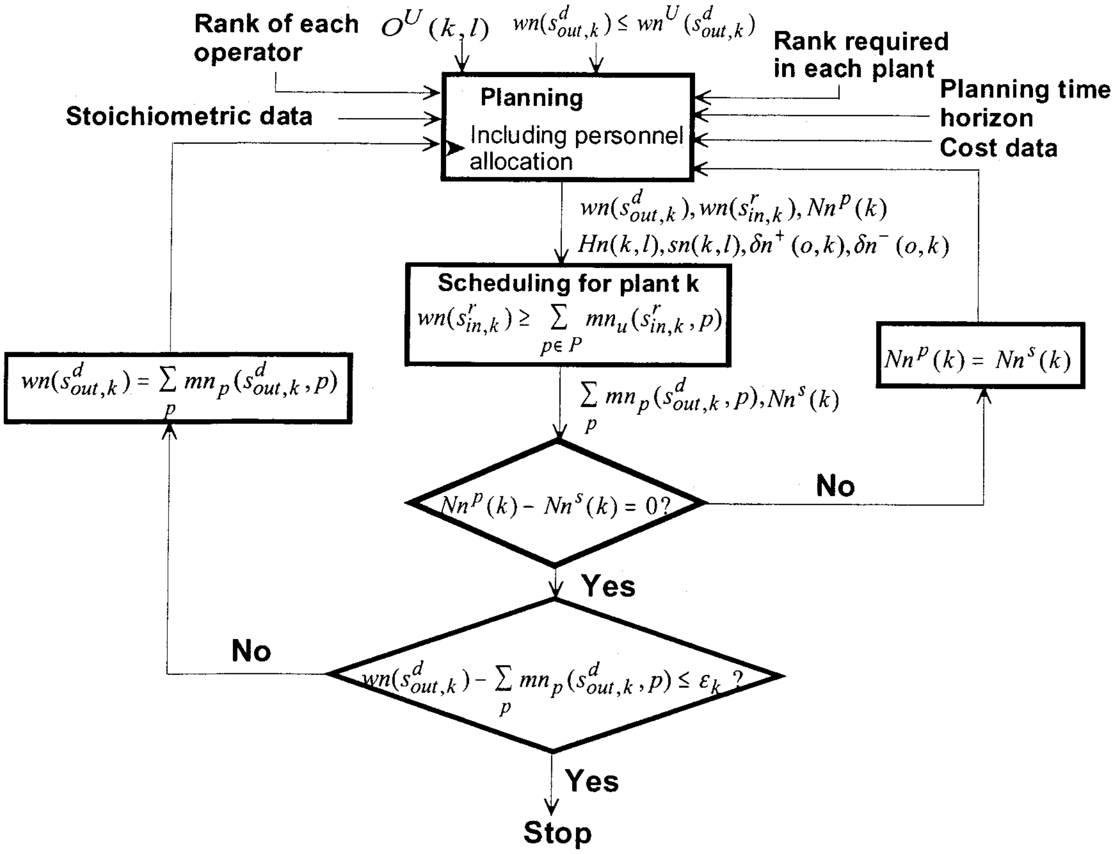

Figure 4. Solution procedure for integrated planning and scheduling problem.

The number of operators required in a three-shift pattern is composed of the operators required in the first ($f1$), the second ($f2$), and the third ($f3$) shifts as shown in Eq. 91. In the first shift, the optimal number of operators is determined by the number of active units before the end of the first shift. The number of operators required in the second shift is determined from

(1) the number of units that begin their operation at or after the end of the first shift and before the end of the second shift [i.e., $Tn(f1, k) \le tn_u(s^*_{in,j}, p) < Tn(f2, k)$], or

(2) the number of units that finish their operation after the end of the first shift and before or at the end of the second shift [i.e., $Tn(f1, k) < tn_p(s_{out,j}, p) \le Tn(f2, k)$].

The number of units that finish their operation after the end of the second shift [i.e., $tn_p(s_{out,j}, p + 1) > Tn(f2, k)$] determines the number of operators required in the third shift

$$
\begin{aligned}
On(k,3) = & \sum_{j\in J_k} A(j) \max_{tn_u(s^*_{in,j},p)<Tn(f1,k)} [yn(s^*_{in,j}, p)] \\
& + \sum_{j\in J_k} A(j) \max_{\substack{Tn(f1,k)\le tn_u(s^*_{in,j},p)<Tn(f2,k) \\ or \\ Tn(f1,k)<tn_p(s_{out,j},p+1)\le Tn(f2,k)}} [yn(s^*_{in,j}, p)] \\
& + \sum_{j\in J_k} A(j) \max_{tn_p(s_{out,j},p+1)>Tn(f2,k)} [yn(s^*_{in,j}, p)] \\
& p \in P_{kn}, \quad k \in K, \quad f1, f2 \in F_3, \quad s^*_{in,j} \in S_{in,j}
\end{aligned} \tag{91}
$$

Therefore, the actual optimal number of operators as determined after the solution of the scheduling model [$Nn^s(k)$] is obtained from the following equation

$$
Nn^s(k) = \sum_{l\in L} On(k, l)sn(k, l) \qquad k \in K \tag{92}
$$

The binary variable supplying the selection of shift patterns [that is, $sn(k, l)$] is obtained from the planning model output, and used in Eq. 92 as a parameter.

Solution Procedure

The solution procedure adopted in this article is based on the methodology developed by Zhu and Majozi (2001), which is based on the fact that a problem of integrated planning and scheduling exhibits a block angular structure. This structure consists of the planning model for the entire site and scheduling submodels corresponding to individual plants. The planning model serves as the coordinator for individual scheduling submodels. The block angular structure for the integrated planning and scheduling problem allows each of the scheduling submodels to be solved independently without compromising global optimality. This eventually leads to much smaller problems with meager binary variables, which can be solved with limited computational effort. This procedure is summarized in Figure 4.

The data shown in Figure 4 correspond to scheduling on a normal day. The same procedure applies to penalized days. Following is the input data to the planning model.

- The rank of each operator and the rank required in each plant. These are obtained from the application of FST as mentioned earlier in this article.
- Stoichiometric data, which are required for mass balances at the planning level.
- The maximum number of operators required in each plant (k), depending on a given shift pattern (l). It is worth noting that the number of operators required in each plant is dependent on the selected shift pattern. A single-shift pattern will almost always require fewer operators than a two-shift pattern. On the same note, a two-shift pattern will required fewer operators than a three-shift pattern.
- The upper bound on the amount of material that can be produced on a particular day (normal or penalized) [i.e., $wn^U(s^d_{out,k})$].
- A chosen planning time horizon. A choice of the planning time horizon determines the number of normal and penalized days. In this article, the planning time horizon has been chosen as one year (52 weeks) with 48 weeks (336 days) allocated to production and 4 weeks reserved for maintenance and contingencies. Given that there are two penalized days (weekend) per week, it implies that 96 days of production will be during penalized days. However, other than weekends, penalized days also include holidays, which are usually known beforehand and vary from one country to another. An allowance of 6 days has been allocated to holidays, which means that over the 48-week planning time horizon, 234 days are normal and 102 days are penalized.
- Cost data, which are required in the objective function for the maximization of profit. These data entail the cost of raw materials, the product selling price, treatment costs, and operator (labor) costs.

Solving the planning model yields the optimal daily production rate [$wn(s^d_{out,k})$] corresponding to an optimal scheduling time horizon for normal and penalized days, and eventually decides the appropriate shift pattern. Concomitant with optimal daily product throughput is the raw material requirement as determined by the stoichiometric constraints. The actual number of operators [$Nn^p(k)$] that accords with the selected shift pattern is also determined. In the first instance, that is, before the scheduling model is solved, this number is determined from the maximum number of operators required in plant (k) in shift pattern (l), as shown in Eqs. 93 and 94. Equation 94 is based on the premise that operator(s) for a particular unit j in plant k will always be available, irrespective of whether the unit is active or inactive

$$Nn^p(k) = \sum_{l \in L} O^{max}(k, l) sn(k, l) \qquad k \in K \tag{93}$$

$$O^{max}(k, l) = l \sum_{j \in J_k} A(j) \qquad l \in L = \{1, 2, 3\}, k \in K \tag{94}$$

To embrace the quality (competency) of personnel within the subsequent scheduling model, the variables that provide for the quality of personnel allocation [$\delta n^+(o, k)$ and $\delta n^-(o, k)$] are determined at the planning level.

The data obtained from the planning model are used as input data to the scheduling model, which is an extension of the model developed by Majozi and Zhu (2001). The fundamental difference between the latter scheduling model and the model presented in this article lies in the duration constraints, as explained earlier. To integrate the planning and scheduling models, the following constraint is incorporated within the scheduling model

$$wn(s^r_{in,j}) \geq \sum_{j \in J_k} \sum_{p \in P_{kn}} mn_u(s_{in,j}, p) \qquad \forall \; s_{in,j} = \text{raw material} \tag{95}$$

The scheduling model is eventually solved to yield the optimal schedule and the maximum possible amount of material that can be produced over the optimal scheduling time horizon determined by the planning model. It should thus be understood that once the time horizon, and hence the shift pattern, has been determined from the planning model, it remains fixed.

The optimal schedule obtained from the solution of the scheduling model is, in essence, a portrait of which units will be active over the scheduling time horizon, when these units will be active, and how much quantity they will have to process. This provides the basis for determination of the optimal number of operators required [$Nn^s(k)$] over the optimal scheduling time horizon as shown in Eq. 92. However, this feat requires a prior knowledge of the number of operators required to operate each unit [that is, $A(j)$]. The number of operators determined at the planning level [$Nn^p(k)$] is then compared to the number of operators predicted at the scheduling level [$Nn^s(k)$]. If these are not equal, the planning level is revisited with a modified number of operators, which is actually the number predicted at the scheduling level. This iterative procedure terminates only when the number of operators at the planning level concurs with that at the scheduling level. This is followed by the next step, which entails the comparison of the amount of daily product predicted at the planning level [$wn(s^d_{out,k})$] with that predicted at the scheduling level [that is, $\sum_p mn_p(s^d_{out,k}, p)$]. If the difference between these amounts exceeds a specified tolerance, the current product throughput from the scheduling model is fed back to the planning model; otherwise, the overall procedure terminates. It is worth noting that the overall procedure terminates when planning model output agrees with the scheduling model output within set tolerances in every respect.

Although the gist of the overall procedure is evident, further elaboration is necessary on the sequence of the termination criteria. Why does consistency on the number of operators predicted at the planning and scheduling levels have to be guaranteed before the corresponding product throughputs, and not vice versa (Figure 4)? The number of operators determined by the initial planning model solution is an upper bound corresponding to a selected shift pattern because the optimal number is not known until the scheduling model has been solved. This eventually reduces the degrees of freedom in the determination of the variables $\delta n^+(s, p)$ and $\delta n^-(s, p)$ because each plant initially requires a large number of operators from a fixed selection pool. This implies that the initial solution of the scheduling model, which is based on these variables, is likely to be suboptimal in terms of the maximum product throughput. Solving the scheduling model yields the optimal number of

Table 1. Summary of the Planning Model

Planning Model*	Initial Solution	Final Solution
NE	11,806	11,806
NBV	618	618
NCV	4942	4942
MILP objective (10^6)	105.10	84.05
Relaxed objective (10^6)	110.41	85.08
CPU time (s)	59.54	0.87
Profit (£(MM)/annum)	70.76	59.15
Labor costs (% operating costs)	7.1	7.0
Sales (£(MM)/annum)	137.15	103.40

*NTP, number of time points; NC, number of constraints; NBV, total number of binary variables; NCV, number of continuous variables.

operators, which is almost always less than the initial number determined at the planning level. Relaying the optimal number of operators back to the planning model improves the degrees of freedom in determining the variables $\delta n^+(s, p)$ and $\delta n^-(s, p)$, thereby improving the subsequent scheduling model. Therefore, guaranteeing consistency of product throughputs between planning and scheduling before the number of operators would certainly culminate in suboptimal results.

Case study

To illustrate the application of this approach, a case study involving three processes from an agrochemical operation is presented. The details of the case study can be found in an article by Majozi and Zhu (2001) and a thesis by Majozi (2001).

Results and Discussion

The results discussed in this section were obtained using GAMS 2.5/CPLEX 7.0 in a 1.5-GHz Pentium IV processor. Tables 1 to 6 give the results obtained after applying the solution strategy presented in Figure 4 of this article. Table 1 summarizes the structure of the planning model and shows initial and final solutions. The initial solution is obtained before integration with scheduling models and the final solution corresponds to the results obtained after integrating the planning and scheduling models. The apparent optimism in terms of profit (£70.76MM/annum) and sales (£137.15MM/annum) in the initial solution is attributed to the fact that the initial planning model is based on the upper bound for the production throughputs, as shown in constraints 9 and 10. On the other hand, the final solution from the integrated planning and scheduling framework takes into account the actual impact of operators on schedules and consequently plant performance, which reflects reality. This explains the relatively low profit and sales of £59.54MM/annum and £103.40MM/annum, respectively.

The planning model consists of 11,806 constraints, 618 binary variables, and 4942 continuous variables, and displays integrality gaps of 4.8 and 1.2% for the initial and final solutions, respectively. This prefigures a very good performance for a model of this size. It is also worth noting that, whereas the initial solution took 59.54 CPU seconds, the subsequent solutions, including the final solution, required an average of 0.9 CPU seconds. It should be mentioned, however, that the overall solution procedure required 145 CPU seconds and 30 iterations. The reduction in CPU time for subsequent iterations emanates from the reduction in the number of variables ascribed to the heuristic of fixing time horizons, hence shift patterns, from the first planning model solution. The foregoing statement is justified by the results shown in Tables 2 and 3. It is clear from these tables that the shift patterns and time horizons determined from the first planning model solution remain fixed throughout the iterative procedure. Plant 1 has to operate only on normal days in a three-shift pattern spanning over a period of 16 h. Plant 2 has to operate on both normal and penalized days in a three-shift pattern spanning over a period of 16 h. Plant 3 has to operate a 12-h single-shift pattern on normal days and a 16-h two-shift pattern on penalized days.

The operator requirement given in Table 2 shows that the optimal number of operators from the initial solution is actually the maximum number of operators corresponding to the selected shift pattern (l) for a particular plant (k) and is obtained from Eq. 93. This operator requirement accounts for 7.1% of the operating cost, which entails raw material, treatment, and labor costs. The utility costs were omitted because the planning model presented in this article does not involve utilities. Table 3 gives the optimal number of operators obtained from the scheduling models. This accounts for 7.0% of the operating cost. Had the number of operators remained fixed from the initial planning model solution, the labor costs would escalate to 10.64%, which justifies the necessity of matching operator requirement to active units over a given time horizon.

Table 4 shows the optimal allocation of operators to different plants. Only 53 operators were selected from the given selection pool of 100 operators. These operators were selected in such a way that each plant is supplied with highly competent (high-grade) operators, while maximizing the overall profit. This explains the omission of most of the operators with very low grades (such as operators 7, 10, 19, and 38), in favor of those with relatively higher grades. The planning model takes into account the financial burden of omitting low-ranking operators in favor of high-ranking operators. It must be mentioned, however, that the performance of the model, with regard to the selection exercise, is highly dependent on the value of the penalty factor (Λ). Increasing this factor leads to the omission of very high ranking in favor of low-ranking operators as a consequence of the financial burden associated with high ranks. It is worth noting that the planning model makes an allowance for some of the operators to work in different plants on normal and penalized days, respectively (see operators 11, 17, 28, and so on). However, in a situation where the plants are far apart this condition has to be removed, as it might be practically infeasible. This is achieved by incorporating an additional constraint into the model.

Table 5 shows the values of the parameters aimed at associating the competency of operators to plant performance. For plant 1, all these values are negative, which characterizes good plant performance. This is attributed to the fact that plant 1 requires operators of grade 12, and all the operators allocated to this plant are of grades that are higher than or equal to 12. The same effect is manifest in plants 2 and 3. It should be noted, however, that the values of these parameters are not equal to the minimum (most negative) values that characterize an ideal scenario, given that the involvement of operator selection and allocation, as well as the impact thereof, introduce practicality into the problem.

Table 6 shows final results from scheduling models of plants 1, 2, and 3. The MILP objective represents the production

Table 2. Initial Planning Model Output

Optimal Output	Plant 1		Plant 2		Plant 3	
	Normal Days	Penalized Days	Normal Days	Penalized Days	Normal Days	Penalized Days
Shift pattern	3	—	3	3	1	2
Time horizon (h)	16.00	—	16.00	16.00	12.00	16.00
Operators required	54	—	21	21	7	14
Product throughput (tons/day)	25.64	—	25.72	1.00	44.51	70.00
Raw Material Requirement (tons/day)						
$s1$	13.85	—	—	—	—	—
$s9$	17.31	—	—	—	—	—
$s10$	24.23	—	—	—	—	—
$s11$	13.85	—	—	—	—	—
$s1'$	—	—	8.47	0.33	—	—
$s2'$	—	—	6.36	0.25	—	—
$s3'$	—	—	4.24	0.17	—	—
$s4'$	—	—	12.71	0.49	—	—
$s5'$	—	—	10.59	0.41	—	—
$s1''$	—	—	—	—	12.02	18.90
$s2''$	—	—	—	—	20.03	31.50
$s3''$	—	—	—	—	8.01	12.00
$s4''$	—	—	—	—	16.02	25.20
$s5''$	—	—	—	—	24.03	37.80
By-product Throughput (tons/day)						
$s7$	17.95	—	—	—	—	—
$s8$	25.64	—	—	—	—	—
$s9'$	—	—	10.29	0.40	—	—
$s11'$	—	—	6.36	0.25	—	—
$s9''$	—	—	—	—	15.58	24.50
$s11''$	—	—	—	—	20.03	31.50

Table 3. Final Planning Model Output

Optimal Output	Plant 1		Plant 2		Plant 3	
	Normal Days	Penalized Days	Normal Days	Penalized Days	Normal Days	Penalized Days
Shift pattern	3	—	3	3	1	2
Time horizon (h)	16.00	—	16.00	16.00	12.00	16.00
Operators required	32	—	14	10	7	11
Product throughput (tons/day)	14.69	—	25.72	1.00	33.33	50.00
Raw Material Requirement (tons/day)						
$s1$	7.93	—	—	—	—	—
$s9$	9.92	—	—	—	—	—
$s10$	13.88	—	—	—	—	—
$s11$	7.93	—	—	—	—	—
$s1'$	—	—	8.47	0.33	—	—
$s2'$	—	—	6.36	0.25	—	—
$s3'$	—	—	4.24	0.17	—	—
$s4'$	—	—	12.71	0.49	—	—
$s5'$	—	—	10.59	0.41	—	—
$s1''$	—	—	—	—	9.00	13.50
$s2''$	—	—	—	—	15.00	22.50
$s3''$	—	—	—	—	6.00	9.00
$s4''$	—	—	—	—	12.00	18.00
$s5''$	—	—	—	—	18.00	27.00
By-product Throughput (tons/day)						
$s7$	10.28	—	—	—	—	—
$s8$	14.69	—	—	—	—	—
$s9'$	—	—	10.29	0.40	—	—
$s11'$	—	—	6.36	0.25	—	—
$s9''$	—	—	—	—	11.67	17.50
$s11''$	—	—	—	—	15.00	22.50

Table 4. Optimal Allocation of Operators to Different Plants

	Plant 1		Plant 2		Plant 3	
Operator	Normal Days	Penalized Days	Normal Days	Penalized Days	Normal Days	Penalized Days
1	×					
2	×					
4	×					
5	×					
6	×					
8	×					
9	×					
11	×			×		
12	×					
13	×					
14	×					
15	×					
17				×	×	
18	×					
20	×					
22	×					
26	×					
28	×			×		
29	×					
30	×					
32	×					
34	×					×
36	×					
39			×	×		
45	×					×
46			×			
48	×			×		
51		×				
52	×			×		
53					×	×
55			×	×		
56			×			
60			×			
61			×			
62	×			×		
67			×			
68	×			×		
71	×					×
72	×			×		
74			×			
75	×					×
76			×			
79	×					×
80			×			
81					×	×
86	×					
88			×			
91					×	×
93					×	×
95					×	×
96					×	×
97			×			
99			×			
Overall	32		14	10	7	11

throughput per day. It is evident from these results that the MILP objective and the number of operators required in each plant on both types of days (that is, normal and penalized) agree very well with the results from the final planning model solution, which is the gist of integrated planning and scheduling. For plant 2, eight and five time points, respectively, were required on normal and penalized days, irrespective of the same length of scheduling time horizon (that is, 16 h). This is attributed to the fact that normal and penalized days have production throughputs of 25.72 and 1.00 tons/day, respectively. Given that the capacity of each equipment in plant 2 is 30 tons, the production throughput of 1.00 ton/day is likely to be unjustifiable in practical terms. To avoid this situation, the lower bounds in Eqs. 11 and 12 would have to be adjusted accordingly. It should also be noted that 7 and 13 time points were used in plant 3 for normal and penalized days, respectively. This is attributed to the fact that, whereas normal days require a time horizon of 12 h, penalized days require a time horizon of 16 h.

To facilitate understanding of how the number of operators

Table 5. Parameter Associating Plant Performance to Operator Allocation

$s^*_{in,j}$	$\rho n^+(s^*_{in,j}, p)$	$\rho n^-(s^*_{ij,j}, p)$	$\rho p^+(s^*_{in,j}, p)$	$\rho p^-(s^*_{in,j}, p)$
		Plant 1		
$s1_{in,1}$	0	−0.204	—	—
$s1_{in,2}$	0	−0.204	—	—
$s2_{in,3}$	0	−0.305	—	—
$s2_{in,4}$	0	−0.305	—	—
$s3_{in,3}$	0	—	—	—
$s3_{in,4}$	0	−0.102	—	—
$s4_{in,5}$	0	−0.102	—	—
$s4_{in,6}$	0	—	—	—
$s4_{in,7}$	0	−0.102	—	—
$s5_{in,8}$	0	−0.305	—	—
$s5_{in,9}$	0	−0.305	—	—
		Plant 2		
$s2'$	0	−0.232	0	−0.203
$s6'$	0	−0.026	0	−0.023
$s7'$	0	−0.052	0	−0.045
$s8'$	0	−0.155	0	−0.135
		Plant 3		
$s2''$	0	−1.350	0	−1.085
$s6''$	0	−0.150	0	−0.121
$s7''$	0	−0.300	0	−0.241
$s8''$	0	−0.900	0	−0.723

is obtained from production schedules, a Gantt chart for plant 2 on penalized days is shown in Figure 5. In this case study, Gantt charts for the production schedules are not shown because their determination was previously presented elsewhere (Majozi and Zhu, 2001), and does not dovetail with the context of this article. According to Tables 3 and 6, plant 2 requires 10 operators and operates in a three-shift pattern spanning over 16 h on penalized days. The encircled numbers in Figure 5 represent the number of operators required to operate a unit. Therefore, 6, 3, and 1 operators are required in the first, second, and third shifts, respectively. This number is mainly attributed to the fact that the second unit starts operating in the first shift and finishes operating in the second shift, which implies that this particular unit is operated by different crews in different shifts.

Conclusions

A novel planning model has been presented in this article. This model determines the optimal scheduling time horizons and shift patterns for individual plants as well as the optimal allocation of operators on normal and penalized days. These optimal scheduling time horizons are then used to perform scheduling for individual plants on normal and penalized days. Scheduling for individual plants is based on the model developed by Majozi and Zhu (2001). Integration of the planning model with the scheduling models for individual plants allows the planning model to determine the optimal number of operators required in each plant on normal and penalized days. Integration of planning and scheduling models is based on the decomposition procedure developed by Zhu and Majozi (2001). Also presented in this article is a procedure for incorporating the quality of operators and their impact on plant performance within the framework of planning and scheduling. The quality of operators is based on their grades or ranks. The latter are obtained by applying fuzzy set theory (FST).

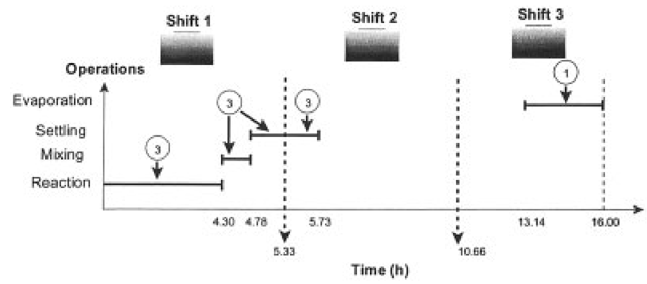

Figure 5. Determination of the optimal number of operators for plant 2 on penalized days.

Application of this planning model to a case study involving three plants and 100 operators showed that, although a significant number of binary variables are required (that is, 618), its structure allows it to be solved with minimal computational effort. A maximum of 59.54 CPU seconds was required in this case study to guarantee global optimality. Global optimality is ensured by linearizing the nonconvex bilinear terms using Glover transformation. These bilinear terms arise from the cost constraints and involve a product of two binary variables or a binary variable and a continuous variable. Also, a maximum integrality gap of 4.8% was realized. This is a manifestation of very good performance for a model of this size. The case study showed that a profit of £70.76 million, which was obtained in the initial planning model solution, was too optimistic when considering the scheduling models. The realistic target was determined to be £59.54 million.

Table 6. Summary of the Scheduling Models after Convergence

Scheduling Model	Plant 1		Plant 2		Plant 3	
	Normal	Penalized	Normal	Penalized	Normal	Penalized
NTP	13	—	8	5	7	13
NC	2475	—	616	385	539	1001
NBV	143	—	32	20	28	52
NCV	1533	—	410	260	390	640
MILP objective (tons/day)	14.50	—	25.72	1.00	33.33	50.00
Relaxed objective	14.50	—	25.72	1.00	33.33	50.00
CPU time (s)	1.92	—	0.11	0.05	0.11	0.22
Operators required	32	—	14	10	7	11

*NTP, number of time points; NC, number of constraints; NBV, total number of binary variables; NCV, number of continuous variables.

Notation

Sets

$F = \{f \mid f \text{ is a shift}\} = \{f1, f2, f3\}$
F_l = a shift belonging to shift pattern l
$J = \{j \mid j \text{ is a unit}\}$
J_k = a unit in process k
$K = \{k \mid k \text{ is a process}\}$
$L = \{l \mid l \text{ is a shift pattern}\} = \{1, 2, 3\}$
$M = \{m \mid m \text{ is a pay category}\} = \{m1, m2, m3, m4\}$
$O = \{o \mid o \text{ is an operator}\}$
$P = \{p \mid p \text{ is a time point}\}$
P_{kn} = a time point corresponding to plant k on a normal day
P_{kp} = a time point corresponding to plant k on a penalized day
$S_{in,\,j} = \{s_{in,\,j} \mid s_{in,\,j} \text{ is an input state to unit } j\}$
$S_{out,\,j} = \{s_{out,\,j} \mid s_{out,\,j} \text{ is an output state to unit } j\}$
$S = \{s \mid s \text{ is any state}\} = S_{in,\,j} \cup S_{out,\,j}$
S_k = a state used in a particular plant k
$s^*_{in,\,j} = \{s^*_{in,\,j} \mid s^*_{in,\,j} \text{ is an effective state}\} \subseteq Sin, j$
$S^r_{in} = \{s^r_{in} \mid s^r_{in} \text{ is a raw material}\} \subseteq S$
$S^r_{in,k} = \{s^r_{in,k} \mid s^r_{in,k} \text{ is a raw material into process } k\} \subseteq S^r_{in} \subseteq S$
$S^d_{out} = \{s^d_{out} \mid s^d_{out} \text{ is a product}\} \subseteq S$
$S^d_{out,k} = \{s^d_{out,k} \mid s^d_{out,k} \text{ is a product from process } k\} \subseteq S^d_{out} \subseteq S$
$S^b_{out} = \{s^b_{out} \mid s^b_{out} \text{ is a by-product}\} \subseteq S$
$S^b_{out,k} = \{s^b_{out,k} \mid s^b_{out,k} \text{ is a by-product from process } k\} \subseteq S^b_{out} \subseteq S$

Variables

$\delta n^+(o, k)$ = slack variable providing for positive deviation between rank required in plant k and the rank of an operator o on a normal day
$\delta p^+(o, k)$ = slack variable providing for positive deviation between rank required in plant k and the rank of an operator o on a penalized day
$\delta n^-(o, k)$ = slack variable providing for negative deviation between rank required in plant k and the rank of an operator o on a normal day
$\delta p^-(o, k)$ = slack variable providing for negative deviation between rank required in plant k and the rank of an operator o on a penalized day
$dn(s, p)$ = amount of state s delivered to customers at time point p on a normal day, $s \in S_{out,\,j}$
$dp(s, p)$ = amount of state s delivered to customers at time point p on a penalized day, $s \in S_{out,\,j}$
$Hn(k, l)$ = scheduling time horizon on a normal day in plant k corresponding to a particular shift pattern l
$Hp(k, l)$ = scheduling time horizon on a penalized day in plant k corresponding to a particular shift pattern l
$mn_p(s, p)$ = amount of state s produced at time point p on a normal day, $s \in S_{out,\,j}$
$mp_p(s, p)$ = amount of state s produced at time point p on a penalized day, $s \in S_{out,\,j}$
$mn_u(s, p)$ = amount of state s used at time point p on a normal day, $s \in S_{in,\,j}$
$mp_u(s, p)$ = amount of state s used at time point p on a penalized day, $s \in S_{in,\,j}$
$Nn^p(k)$ = number of operators required in plant k on a normal day from the planning model
$Np^p(k)$ = number of operators required in plant k on a penalized day from the planning model
$Ovtn(k)$ = overtime in plant k on a normal day
$Ovtp(k)$ = overtime in plant k on a penalized day
$qn_s(s, p)$ = amount of state s stored at time point p on a normal day
$qp_s(s, p)$ = amount of state s stored at time point p on a penalized day
$sn(k, l)$ = binary variable associated with choice of shift pattern l in plant k on a normal day
$sp(k, l)$ = binary variable associated with choice of shift pattern l in plant k on a penalized day
$SPC(k, l)$ = costs associated with a particular shift pattern l in plant k
$tn_p(s, p)$ = time at which state s is produced at time point p on a normal day, $s \in S_{out,\,j}$
$tp_p(s, p)$ = time at which state s is produced at time point p on a penalized day, $s \in S_{out,\,j}$
$tn_u(s, p)$ = time at which state s is used at time point p on a normal day, $s \in S_{in,\,j}$
$tp_u(s, p)$ = time at which state s is used at time point p on a penalized day, $s \in S_{in,\,j}$
$tn_a(s)$ = actual processing time for state s on a normal day, $s \in S_{in,\,j}$
$tp_a(s)$ = actual processing time for state s on a penalized day, $s \in S_{in,\,j}$
$Varn(k, l)$ = time beyond the minimum length of shift pattern l in plant k on a normal day
$Varp(k, l)$ = time beyond the minimum length of shift pattern l in plant k on a penalized day
$W(s^r_{in})$ = overall amount of raw material s^r_{in} used over the planning time horizon
$w(s^r_{in,k})$ = amount of raw material s^r_{in} used in process k over the planning time horizon
$wn(s^r_{in,k})$ = amount of raw material s^r_{in} used in process k on a normal day
$wp(s^r_{in,k})$ = amount of raw material s^r_{in} used in process k on a penalized day
$W'(s^r_{in})$ = additional amount of raw material s^r_{in} available at a penalty over the planning time horizon
$W(s^b_{out})$ = overall amount of by-product s^b_{out} produced over the planning time horizon
$w(s^b_{out,k})$ = amount of by-product s^b_{out} produced in process k over the planning time horizon
$wn(s^b_{out,k})$ = amount of by-product s^b_{out} produced in process k on a normal day
$wp(s^b_{out,k})$ = amount of by-product s^b_{out} produced in process k on a penalized day
$W'(s^b_{out})$ = additional amount of by-product s^b_{out} treated at a penalized cost over the planning time horizon
$w(s^d_{out,k})$ = amount of product s^d_{out} produced in process k over the planning time horizon
$wn(s^d_{out,k})$ = amount of product s^d_{out} produced in process k on a normal day
$wn^U(s^d_{out,k})$ = upper bound on the amount of product s^d_{out} produced in process k on a normal day
$wn^L(s^d_{out,k})$ = lower bound on the amount of product s^d_{out} produced in process k on a normal day
$wp(s^d_{out,k})$ = amount of product s^d_{out} produced in process k on a penalized day
$wp^U(s^d_{out,k})$ = upper bound on the amount of product s^d_{out} produced in process k on a penalized day
$wp^L(s^d_{out,k})$ = lower bound on the amount of product s^d_{out} produced in process k on a penalized day
$xn(k, o)$ = binary variable associated with the allocation of operator o to plant k on a normal day
$xp(k, o)$ = binary variable associated with the allocation of operator o to plant k on a penalized day
$yn(s, p)$ = binary variable associated with usage of state s at time point p on a normal day, $s \in S^*_{in,\,j}$
$yp(\mathrm{s, p})$ = binary variable associated with usage of state s at time point p on a penalized day, $s \in S^*_{in,\,j}$

Parameters

$\alpha(s^r_{in,k})$ = stoichiometric coefficient of raw material s^r_{in} when used in plant k
$A(j)$ = number of operators required to operate a particular unit j
$\beta(s^b_{out,k})$ = stoichiometric coefficient of by-product s^b_{out} when produced in plant k
$B(s^b_{out})$ = maximum amount of by-product s^b_{out} treated at standard cost over the planning time horizon

$B'(s^b_{out})$ = maximum penalized amount of by-product s^b_{out} over the planning time horizon
$C(s^d_{out})$ = selling price per unit capacity for product s^d_{out}
$C(s^r_{in})$ = standard price per unit capacity for raw material s^r_{in}
$C(s^b_{out})$ = standard treatment cost per unit capacity for by-product s^b_{out}
$C'(s^r_{in})$ = penalized price per unit capacity for raw material s^r_{in}
$C'(s^b_{out})$ = penalized treatment cost per unit capacity for by-product s^b_{out}
$\Delta n^+(k), \Delta n^-(k)$ = positive and negative slack variable ratios in plant k on a normal day
$\Delta p^+(k), \Delta p^-(k)$ = positive and negative slack variable ratios in plant k on a penalized day
$Dn(k)$ = number of normal days in plant k over the planning time horizon
$Dp(k)$ = number of penalized days in plant k over the planning time horizon
$G^L(o)$ = minimum rank (grade) for all operators
$G^U(o)$ = maximum rank (grade) for all operators
$G(k)$ = rank (grade) required in plant k obtained from FST application
$G'(o)$ = rank (grade) of a particular operator o obtained from FST application
$Hn^L(l)$ = lower bound on normal day scheduling time horizon corresponding to a particular shift pattern l
$Hp^L(l)$ = lower bound on penalized day scheduling time horizon corresponding to a particular shift pattern l
$Hn^U(l)$ = upper bound on normal day scheduling time horizon corresponding to a particular shift pattern l
$Hp^U(l)$ = upper bound on penalized day scheduling time horizon corresponding to a particular shift pattern l
Hn^{ov} = time period beyond which work is classified as overtime on a normal day
Hp^{ov} = time period beyond which work is classified as overtime on a penalized day
Λ = penalty term providing for the minimization of deviation between the rank required by the plant and the rank of an operator
$\upsilon(s)$ = allowed percentage time variation for processing state s, $s \in S_{in,j}$
$Nn^s(k)$ = number of operators required in plant k on a normal day from the scheduling model
$Np^s(k)$ = number of operators required in plant k on a penalized day from the scheduling model
$On(k, l)$ = number of operators required in plant k on a normal day for shift pattern l
$Op(k, l)$ = number of operators required in plant k on a penalized day for shift pattern l
$O^{max}(k, l)$ = maximum number of operators required in plant k for a shift pattern l
$Q^0_s(s)$ = initial amount of state s stored
$\rho p^+(s, p), \rho p^-(s, p)$ = parameters determined by the number and allocation of operators to a particular plant on a penalized day, $s \in S_{in,j}$
$\rho n^+(s, p), \rho n^-(s, p)$ = parameters determined by the number and allocation of operators to a particular plant on a normal day, $s \in S_{in,j}$
$R(s^r_{in})$ = maximum amount of raw material s^r_{in} available at standard price
$R'(s^r_{in})$ = maximum penalized amount of raw material s^r_{in} over planning time horizon
$Rem(o, m)$ = hourly remuneration of operator o corresponding to pay category m
$Rem^L(m)$ = minimum remuneration corresponding to a particular pay category m
$Rem^U(m)$ = maximum remuneration corresponding to a particular pay category m
$Rem(o)$ = remuneration of a particular operator o
Rem^L = lower bound on remuneration
Rem^U = upper bound on remuneration
$\tau(s)$ = mean processing time for state s, $s \in S_{in,j}$
$Tn(f, k)$ = end time of a particular shift f in plant k on a normal day
$Tp(f, k)$ = end time of a particular shift f in plant k on a penalized day
V_j = capacity of a particular unit j
$wn^L(s^d_{out,k})$ = lower bound on the amount of product s^d_{out} produced in process k on a normal day
$w^U(s^d_{out,k})$ = maximum amount of product s^d_{out} produced in process k over the planning time horizon

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